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Some $q$-analogues of the SchrÃ $\|$ der numbers arising from combinatorial statistics on lattice paths

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## Abstract

We present enumerative results concerning plane lattice paths starting at the origin, with steps $(1,0),(1,1)$ and $(0,1)$. Such paths with a specified endpoint are counted by the Delannoy numbers, while those paths which in addition do not run above the line $y=x$ are counted by the SchrÃ $\|$ der numbers. We develop $q$-analogues of the Delannoy and SchrÃ \|der numbers derived from several combinatorial statistics: the number of diagonal steps, the area under the path, and the major index. We investigate the symmetry and unimodality of the resulting polynomials, and determine the asymptotic behavior of the expected number of diagonal steps and area under a path. Using the number of diagonal steps statistic, we describe the $Æ^{\prime}$-vector of the associahedron in terms of lattice paths counted by the SchrÃ $\|$ der numbers.

## MSC

05A99

## Keywords

Associahedron; Catalan; combinatorial statistics; Delannoy; Æ્'-vector; $h$-vector; inversion; lattice path; major index; Narayana; non-crossing partition; polytope; $q$-analog; SchrÃ \|der; symmetry; unimodality

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