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A unified framework for the construction of one-step finite volume and discontinuous Galerkin schemes on unstructured meshes

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Abstract

In this article, a conservative least-squares polynomial reconstruction operator is applied to the discontinuous Galerkin method. In a first instance, piecewise polynomials of degree N are used as test functions as well as to represent the data in each element at the beginning of a time step. The time evolution of these data and the flux computation, however, are then done with a different set of piecewise polynomials of degree Mâ©¾N, which are reconstructed from the underlying polynomials of degree N. This approach yields a general, unified framework that contains as two special cases classical high order finite volume (FV) schemes (N=0) as well as the usual discontinuous Galerkin (DG) method (N=M). In the first case, the polynomial is reconstructed from cell averages, for the latter, the reconstruction reduces to the identity operator. A averages, for the latter, the reconstruction reduces to the identity operator. A completely new class of numerical schemes is generated by choosing Nâ‰ 0 and M>N. The reconstruction operator is implemented for arbitrary polynomial degrees N and M on unstructured triangular and tetrahedral meshes in two and three space dimensions.

To provide a high order accurate one-step time integration of the same formal order of accuracy as the spatial discretization operator, the (reconstructed) polynomial data of degree M are evolved in time locally inside each element using a new *local* continuous spaceâ€“time Galerkin method. As a result of this approach, we obtain, as a high order accurate predictor, spaceâ€“time polynomials for the vector of conserved variables and for the physical fluxes and source terms, which then can be used in a natural way to construct very efficient *fully-discrete* and *quadrature-free* one-step schemes. This feature is particularly important for DG schemes in three space dimensions, where the cost of numerical quadrature may become prohibitively expensive for very high orders of accuracy.

Numerical convergence studies of all members of the new general class of proposed schemes are shown up to sixth-order of accuracy in space *and time* on unstructured two- and three-dimensional meshes for two very prominent nonlinear hyperbolic systems, namely for the Euler equations of compressible gas dynamics and the equations of ideal magnetohydrodynamics (MHD). The results indicate that the new class of intermediate schemes (Nâ‰ 0,M>N) is computationally more efficient than classical finite volume or DG schemes.

Finally, a large set of interesting test cases is solved on unstructured meshes, where the proposed new time stepping approach is applied to the equations of ideal and relativistic MHD as well as to nonlinear elasticity, using a standard high order WENO finite volume discretization in space to cope with discontinuous solutions.

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Keywords

Hyperbolic PDE; Unstructured meshes; Finite volume; Discontinuous Galerkin; M-exact PNPM least squares reconstruction; One-step time discretization; Local continuous spaceâ€“time Galerkin method; WENO; Euler equations; Ideal and relativistic MHD equations; Nonlinear elasticity

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