REVIEWS OF MODERN PHYSICS

# Phase and Angle Variables in Quantum Mechanics

P. CARRUTHERS and MICHAEL MARTIN NIETO

Rev. Mod. Phys. **40**, 411 – Published 1 April 1968



## ABSTRACT

The quantum-mechanical description of phase and angle variables is reviewed, with emphasis on the proper mathematical description of these coordinates. The relations among the operators and state vectors under consideration are clarified in the context of the Heisenberg uncertainty relations. The familiar case of the azimuthal angle variable ■ and its "conjugate" angular momentum Lz is discussed. Various pitfalls associated with the periodicity problem are avoided by employing periodic variables (sin■ and cos■ to describe the phase variable. Well-defined uncertainty relations are derived and discussed. A detailed analysis of the three-dimensional harmonic oscillator excited in coherent states is given. A detailed analysis of the simple harmonic oscillator is given. The usual assumption that a (Hermitian) phase operator ■ (conjugate to the number operator N) exists [illegible]n to be erroneous. However, cosine and sine operators C and S exist [illegible] the appropriate phase variables. A Poisson bracket argument using action-angle (rather J, cos , sin ) variables is used to deduce C and S. The spectra and eigenfunctions of these operators are investigated, along with the important "phase-difference" periodic variables. The properties of the oscillator variables in the various types of states are analyzed with special attention to the uncertainty relations and the transition to the classical limit. The utility of coherent states as a basis for the description of the evolution of the density matrix is emphasized. In this basis it is easy to identify the classical Liouville equation in action-angle variables along with quantum-mechanical "corrections." Mention is made of possible physical applications to superfluid systems.

DOI: https://doi.org/10.1103/RevModPhys.40.411



## AUTHORS & AFFILIATIONS

P. CARRUTHERS

Laboratory of Nuclear Studies and Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York

MICHAEL MARTIN NIETO

Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York

## REFERENCES (SUBSCRIPTION REQUIRED)

CLICK TO EXPAND

Issue

Vol. 40, Iss. 2 — April - June 1968

ISSN 1539-0756 (online), 0034-6861 (print). 

Phase and angle variables in quantum mechanics, .
Phase in Optics,
Quantum optics,
Phase in quantum optics,
Newton-Leibniz integration for ket-bra operators in quantum mechanics and derivation of entangled state representations,
Modern quantum mechanics, revised edition,
Quantum phase and phase dependent measurements,
Advances in atomic physics: an overview,
Quantum statistical mechanics,