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# Phase and Angle Variables in Quantum Mechanics

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## ABSTRACT

The quantum-mechanical description of phase and angle variables is reviewed, with emphasis on the proper mathematical description of these coordinates. The relations among the operators and state vectors under consideration are clarified in the context of the Heisenberg uncertainty relations. The familiar case of the azimuthal angle variable  $\phi$  and its "conjugate" angular momentum  $L_z$  is discussed. Various pitfalls associated with the periodicity problem are avoided by employing periodic variables ( $\sin\phi$  and  $\cos\phi$ ) to describe the phase variable. Well-defined uncertainty relations are derived and discussed. A detailed analysis of the three-dimensional harmonic oscillator excited in coherent states is given. A

detailed analysis of the simple harmonic oscillator is given. The usual assumption that a (Hermitian) phase operator  $\hat{\phi}$  (conjugate to the number operator  $N$ ) exists

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is to be erroneous. However, cosine and sine operators  $C$  and  $S$  exist with the appropriate phase variables. A Poisson bracket argument using

action-angle (rather  $J, \cos \theta, \sin \theta$ ) variables is used to deduce  $C$  and  $S$ . The spectra and eigenfunctions of these operators are investigated, along with the important "phase-difference" periodic variables. The properties of the oscillator variables in the various types of states are analyzed with special attention to the uncertainty relations and the transition to the classical limit. The utility of coherent states as a basis for the description of the evolution of the density matrix is emphasized. In this basis it is easy to identify the classical Liouville equation in action-angle variables along with quantum-mechanical "corrections." Mention is made of possible physical applications to superfluid systems.

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